

I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2018
MATHEMATICS-III

(Com. to CE,CSE,IT,AE,AME,EIE,EEE,ME,ECE,Metal E,Min E,E Com E,Agri E,Chem E,PCE,PE)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Write the working procedure to reduce the given matrix into Echelon form. (2M)
- b) Find the Eigen value of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$. (2M)
- c) Find the point of the curve $r = a(1 + \cos \theta)$ where tangent coincide with the radius vector. (2M)
- d) Evaluate $\int_1^2 \int_3^4 (xy + e^y) dx dy$ (2M)
- e) Show that $\Gamma(n+1) = n\Gamma(n)$ for $n > 0$ (2M)
- f) Find grad ϕ where $\phi = x^3 + y^3 + 3xyz$ at $(1,1,-2)$ (2M)
- g) Find the work done in moving particle in the force field $\vec{F} = 3x^2 \vec{i} + \vec{j} + z\vec{k}$ along the straight line $(0, 0, 0)$ to $(2, 1, 3)$. (2M)

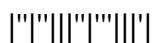
PART -B

2. a) Reduce the matrix $A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ in to normal form hence find the rank. (7M)
- b) If consistent, solve the system of equations. (7M)

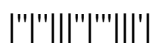
$$\begin{aligned} x + y + z + t &= 4 \\ x - z + 2t &= 2 \\ y + z - 3t &= -1 \\ x + 2y - z + t &= 3. \end{aligned}$$
3. a) Determine the diagonal matrix orthogonally similar to the matrix. (7M)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
- b) Find the Nature, index and signature of the quadratic form (7M)

$$10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$$



4. a) By change of order of integration evaluate $\int_0^a \int_x^a (x^2 + y^2) dy dx$ (7M)
- b) Evaluate $\int_0^{\pi/2} \int_0^{a \sin \theta} \int_0^{(a^2 - r^2)/a} r dr d\theta dz$ (7M)
5. a) Evaluate $\int_0^{\infty} 3^{-4x^2} dx$ (7M)
- b) Show that $\int_0^{\infty} \sin x^2 dx = \int_0^{\infty} \cos x^2 dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$ (7M)
6. a) Show that $\bar{f} = r^n (\bar{a} \times \bar{r})$ is solenoidal where $\bar{a} = a_1 \bar{i} + a_2 \bar{j} + a_3 \bar{k}$ and $\bar{r} = x \bar{i} + y \bar{j} + z \bar{k}$ (7M)
- b) Prove that $\nabla \left(r \nabla \left(\frac{1}{r^3} \right) \right) = \frac{3}{r^4}$ (7M)
7. a) Verify stoke's theorem for $\bar{F} = y \bar{i} + z \bar{j} + x \bar{k}$ for the upper part of the sphere $x^2 + y^2 + z^2 = 1$. (7M)
- b) Verify Green's theorem in the plane for $\oint_c (xy + y^2) dx + x^2 dy$. Where c is the closed curve of the region bounded by $y=x$ & $y=x^2$ (7M)



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PART -A

1. a) Write the working procedure to reduce the given matrix into Normal form. (2M)
- b) Write quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ (2M)
- c) Write the tangents at the origin of the curve $a^2y^2 = x^2(a^2 - x^2)$. (2M)
- d) Evaluate $\int_0^1 \int_0^1 \int_0^1 dx dy dz$ (2M)
- e) Prove that $\beta(m, n) = \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ (2M)
- f) Find the maximum value of the directional derivative of $\phi = 2x^2 - y - z^4$ at $(2, -1, 1)$ (2M)
- g) Write Stoke's theorem. (2M)

PART -B

2. a) For what value of k the matrix $A = \begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank 3. (7M)

$$8x - 3y + 2z = 20$$

- b) Solve the following system of equations $4x + 11y - z = 33$ by using. (7M)

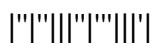
$$6x + 3y + 12z = 35$$

Gauss – Seidel method.

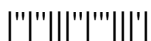
3. a) Determine the characteristic roots and the corresponding characteristic vectors of the matrix. (7M)

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

- b) Find the Nature, index and signature of the quadratic form $4x^2 + 3y^2 + z^2 - 8xy + 4xz - 6yz$ (7M)



4. a) Trace the curve $r^2 = a^2 \cos 2\theta$ (7M)
- b) Evaluate $\int \int (x^2 + y^2) dx dy$ over the area bounded by the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (7M)
5. a) Evaluate $\int_0^{\infty} a^{-bx^2} dx$ $b > 0, a > 1$ (7M)
- b) Show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ (7M)
6. a) Find the constants 'a' and 'b' such that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ cuts orthogonally at $(1, -1, 2)$ (7M)
- b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. (7M)
7. a) If $\vec{f} = (3x^2 - 2z)\bar{i} - 4xy\bar{j} - 5x\bar{k}$ Evaluate $\int_V \text{Cur } \vec{F} dv$, where v is volume bounded by the planes $x = 0; y = 0; z = 0$ and $3x + 2y - 3z = 6$. (7M)
- b) Evaluate $\oint_C \cos y dx + x(1 - \sin y) dy$ over a closed curve c given by $x^2 + y^2 = 1; z = 0$ using Green's theorem. (7M)



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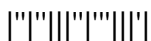
PART -A

1. a) Write the working procedure to find the inverse of the given matrix by Jordan method. (2M)
- b) Find the Eigen value of Adj A if the 'λ' is the Eigen value of A. (2M)
- c) Write the symmetry of the curve $y^2(2a-x) = x^3$ (2M)
- d) Evaluate $\int_{0-x}^x \int xy dx dy$ (2M)
- e) Find the value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ (2M)
- f) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (2M)
- g) Write the physical interpretation of Gauss divergence theorem. (2M)

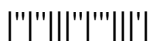
PART -B

2. a) Reduce the matrix to Echelon form and find its rank (7M)

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$
- b) Solve the equations $10x + y + z = 12$,
 $2x + 10y + z = 13$, by Gauss – Jordan method. (7M)
 $x + y + 5z = 7$.
3. a) Find the Natural frequencies and normal modes of vibrating system for which (7M)
 mass $M = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $K = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$
- b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Hence find A^{-1} (7M)



4. a) Find the volume of region bounded by the surface $z = x^2 + y^2$ and $z = 2x$. (7M)
- b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy dx$ by changing in to polar co-ordinates. (7M)
5. a) Show that $\int_a^b (x-a)^{m-1} (b-x)^{n-1} dx = (b-a)^{m+n-1} \beta(m,n) m > 0, n > 0$ (7M)
- b) Evaluate $\int_0^1 (x \log x)^4 dx$ (7M)
6. a) Find the directional derivative of $\phi = xyz$ at $(1, -1, 1)$ along the direction which makes equal angles with the positive direction of x, y, z axes (7M)
- b) Prove that $\text{div curl } \vec{f} = 0$ (7M)
7. a) Verify Green's theorem for $\int_c (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where c is the boundary of the region enclosed by the lines. $x = 0, y = 0, x + y = 1$. (7M)
- b) Find the flux of vector function $\vec{F} = (x - 2z)\vec{i} + (x + 3y)\vec{j} + (5x + y)\vec{k}$ through the upper side of the triangle ABC with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$. (7M)



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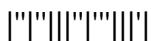
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PART -A

1. a) Find the Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (2M)
- b) Prove the AB and BA has same Eigen values. (2M)
- c) Write the Asymptote of the curve $y = \frac{x^2 + 1}{x^2 - 1}$ (2M)
- d) Evaluate $\int_0^3 \int_1^2 xy(x+y) dx dy$ (2M)
- e) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (2M)
- f) Show that $\nabla(r^2) = 2\vec{r}$ (2M)
- g) Write Green's theorem. (2M)

PART -B

2. a) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$ into PAQ form and hence find the rank (7M)
 of the matrix.
- b) Solve the equations $x + y + z = 8,$
 $2x + 3y + 2z = 19$ by Gauss – Elimination method. (7M)
 $4x + 2y + 3z = 23$
3. a) Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ if possible. (7M)
- b) Find the Nature, index and signature of the quadratic form (7M)
 $3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ by orthogonal reduction.



4. a) Trace the curve $x = a \cos t + \frac{a}{2} \log \tan^2 t/2$, $y = a \sin t$ (7M)
- b) Find the area between the circles $r = a \cos \theta$ and $r = 2a \cos \theta$. (7M)
5. a) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{\beta(m,n)}{(a+b)^m a^n}$ (7M)
- b) Evaluate $\int_0^{\infty} e^{-x^6} x^4 dx$ (7M)
6. a) Find the directional derivative of the function $e^{2x} \cos yz$ at the origin in the direction to the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \frac{\pi}{4}$ (7M)
- b) Show that $\text{curl curl } \bar{f} = \nabla \times (\nabla \times \bar{f}) = \nabla(\nabla \cdot \bar{f}) - (\nabla \cdot \nabla) \bar{f}$ if $\bar{f}(x, y, z)$ is vector point function. (7M)
7. a) Verify Gauss Divergence theorem for $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a$; $0 \leq y \leq b$; $0 \leq z \leq c$. (7M)
- b) Evaluate $\iint_s (\nabla \times \bar{F}) \cdot \bar{n} ds$ where $\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xy + z^2)\bar{k}$ and s in the surface of the paraboloid $z = 4 - x^2 - y^2$ above the xy plane. (7M)

