

II B. Tech I Semester Regular Examinations, October/November - 2017
RANDOM VARIABLES & STOCHASTIC PROCESSES
 (Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**

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PART -A

1. a) What are the conditions for a function to be a random variable? [2M]
- b) Define various types of transformation of Random variables. [2M]
- c) Show that  $\text{var}(X+Y) = \text{var}(x)+\text{var}(Y)$ , if X&Y are statistical independent random variables. [3M]
- d) Differentiate between Random Processes and Random variables with an example [2M]
- e) If the Auto correlation function of wide sense stationary X(t) is  $R_{XX}(\tau)=4+2e^{-2\tau}$ . Find the area enclosed by the power spectrum density curve of X(t). [3M]
- f) Define linear system and write the expression for output response. [2M]

PART -B

2. a) Write the properties of Gaussian density curve. Find the maximum value of Gaussian density function [8M]
- b) A random variable X has the distribution function [6M]

$$F_X(x) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n)$$

Find the probability of a)  $P\{-\infty < X \leq 6.5\}$  b)  $p\{X > 4\}$  c)  $p\{6 < X \leq 9\}$

3. a) State and prove the chebychev's inequality theorem? [7M]
  - b) If X is a discrete random variable with a Moment generating function of  $M_X(0)$ , find the Moment generating function of [7M]
- i)  $Y=aX+b$  ii)  $Y=KX$  iii)  $Y=\frac{X+a}{b}$

4. a) State and explain the properties of joint density function [7M]
- b) State and prove the central limit theorem. [7M]

5. a) Explain the following [7M]
- i) Stationarity ii) Ergodicity  
 iii) Statistical independence with respect to random processes
- b) A random process is given as  $X(t) = At$ , where A is a uniformly distributed random variable on (0,2). Find whether X(t) is wide sense stationary or not. [7M]

6. a) The power spectral density of a stationary random process is given by [7M]
- $$S_{XX}(w) = \begin{cases} A & -k < w < k \\ 0 & \text{otherwise} \end{cases}$$

Find the auto correlation function.

- b) Derive the relationship between cross-power spectral density and cross correlation function. [7M]



7. a) The input to an LTI system with impulse response  $h(t) = \delta t + t^2 e^{-at}$ .  $U(t)$  is a WSS [7M] process with mean of 3. Find the mean of the output of the system.
- b) Write Short notes on Mean and Mean Squared Value of System response of a linear [7M] system.



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**PART -A**

1. a) Give the classical axiomatic definitions of Probability [2M]
- b) Find the variance of X for uniform density function. [3M]
- c) The joint probability density function of X&Y is [3M]
 
$$f_{X,Y}(x,y) = \begin{cases} c(2x+y); & 0 \leq x \leq 2, 0 \leq y \leq 3 \\ 0; & \text{else} \end{cases}$$

Then find the value of constant c.
- d) Prove that the Auto correlation function has maximum value at the origin i.e. [2M]  
 $|R_{XX}(\tau)| = R_{XX}(0)$
- e) If the Power spectrum density of x(t) is  $S_{XX}(\omega)$ , find the PSD of  $\frac{dx(t)}{dt}$  [2M]
- f) Define Effective Noise temperature and Average Noise Figure [2M]

**PART -B**

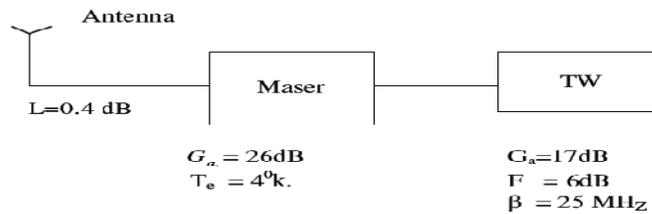
2. a) Define and explain the properties of CDF function of a random variable. [8M]
- b) Assume that X is a continuous random variable with the following pdf: [6M]
 
$$f_X(x) = f(x) = \begin{cases} A(2x - x^2), & 0 < x < 2 \\ 0, & \text{Otherwise} \end{cases}$$

i) What is the Value of A? ii) Find  $P[X > 1]$  iii) Evaluate CDF function.
3. a) Find the Moment generating function of exponential distribution? [6M]
- b) A continuous distribution is given by [8M]
 
$$F_X(x) = \begin{cases} \frac{1}{x\sqrt{2\pi}} e^{-(\log x)^2/2} & x > 0 \\ 0 & x < 0 \end{cases}$$

Find the mean, standard deviation and co-efficient of skewness of this distribution
4. a) State and prove any three properties of joint characteristic function. [6M]
- b) Random variables X and Y have a Joint Probability density function given by [8M]
 
$$f_{XY}(x, y) = \begin{cases} \frac{1}{\pi} & \text{for } x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine whether random variables x and y are  
 i) Statistically independent ii) Uncorrelated.
5. a) Explain about the following random processes [6M]
  - i) Mean ergodic process ii) Correlation ergodic process iii) Gaussian random process
- b) Prove that the random process  $X(t) = A \cos(\omega t + \phi)$  is wide sense stationary if it is assumed that  $\omega c$  is a constant and  $\phi$  is uniformly distributed variable in the interval  $(0, 2\pi)$ . [8M]

6. a) Calculate the power Spectral Density of a stationary random process for which [7M]  
the Autocorrelation is  $R_{xx}(\tau) = \sigma^2 e^{-\alpha|\tau|}$
- b) State and prove the relationship between Power Density Spectrum and [7M]  
Autocorrelation Function
7. a) For the following receiver, calculate the effective noise temperature. [6M]



- b) Derive the relation between input PSD and output PSD of an LTI system [8M]



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**PART -A**

1. a) How you can differentiate mutually exclusive events and independent event? [3M]
- b) Define skew and skewness [2M]
- c) Define Marginal Distribution Functions [2M]
- d) List out the properties of Autocorrelation Function [3M]
- e) Define Wide Sense Stationary Process and write its conditions [2M]
- f) Write the expression for average noise figure of cascaded networks. [2M]

**PART -B**

2. a) In a binary communication system, the probability of bit error is 0.01. If a block of 8 bits are transmitted, find the probability that [8M]
  - (a) Exactly 2 bit errors will occur
  - (b) At least 2 bit errors will occur
  - (c) More than 2 bit errors will occur
  - (d) All the bits will be erroneous
- b) For real constants  $b > 0$ ,  $c > 0$  and any  $a$ , find condition on constant  $a$  and relationship between  $a$  and  $c$  (for given  $b$ ) such that the function is a valid probability density [6M]

$$f_X(x) = \begin{cases} a(1 - (x/b)) & 0 \leq x \leq c \\ 0 & \text{else where} \end{cases}$$

3. a) Find the Mean and Variance of the Poisson RV [7M]
- b) Explain Nonmonotonic Transformations of Continuous Random Variable [7M]
4. a) If  $X$  and  $Y$  are independent, show that  $E[XY] = E[X] E[Y]$ . [7M]
- b) The joint density function of two random variables  $X$  and  $Y$  is [7M]

$$f_{XY}(x, y) = \begin{cases} \frac{(x+y)^2}{40} & ; -1 < x < 1 \text{ and } -3 < y < 3 \\ 0 & ; \text{otherwise} \end{cases}$$

Find the variances of  $X$  and  $Y$ 

5. a) Define a random process. Write the classification of random process by the form of its sample functions and explain [7M]
- b) Autocorrelation function of an ergodic stationary random process with no periodic component is given as  $100 + 4/(1 + 6\tau^2)$ . Find the mean and variance of the process. [7M]

6. a) Prove that Power Density Spectrum and Autocorrelation Function forms a Fourier transform pair [7M]  
b) A random process  $Y(t)$  has the power spectral density [7M]

$$S_{YY}(\omega) = \frac{9}{\omega^2 + 64}$$

- Find
- The average power of the process
  - The Auto correlation function

7. a) Derive the equivalent Effective noise temperature of a cascaded network. [7M]  
b) Explain about Band limited random process and its properties. [7M]



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**PART -A**

1. a) State Bayes' Theorem [2M]
- b) What are the properties of Gaussian Random variable? [3M]
- c) State Central limit theorem [2M]
- d) Define a random process. Write the classification of random process by the form of its sample functions [3M]
- e) Explain about Statistical independence with respect to random processes [2M]
- f) What is the Mean-squared value of a linear system? [2M]

**PART -B**

2. a) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball? [7M]
- b) Write the properties of Poisson distribution and density curve. Find the maximum value of Poisson density function [7M]
3. a) Explain about Monotonic Transformations for a Continuous Random Variable [7M]
- b) Find the Moment generating function of exponential distribution? [7M]
4. a) Explain the conditions to be satisfied by an ergodic random process [7M]
- b) Two statistically independent random variables X and Y have respective densities  $f_X(x) = 5 e^{-5x} u(x)$ ,  $f_Y(y) = 2 e^{-2y} u(y)$ . Find the density of the sum  $W=X+Y$ . [7M]
5. a) Write all the properties of joint density function. [6M]
- b) A joint probability density function is [8M]
 
$$f_{xy}(x,y) = \begin{cases} \frac{1}{24} & 0 < x < 6, 0 < y < 4 \\ 0 & \text{else where} \end{cases}$$
 Find the expected value of the function  $g(X,Y) = (XY)^2$
6. a) Determine the cross-correlation function corresponding to the cross power spectrum  $S_{XY}(\omega) = 8/(\alpha + j\beta)^3$ , where  $\alpha$  is a constant [7M]
- b) Derive the expression for power spectral density of a random process [7M]



7. a) Prove that

[7M]

$$F = F_1 + \frac{F_2 - 1}{g_{a_1}} + \frac{F_3 - z}{g_{a_1} g_{a_2}} + \dots$$

for cascade of two-port networks

b) Write notes on modelling of thermal noise source

[7M]

